ASSIGNMENT
CH-MATRICES

Q1. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$**, find *x,y,z,w*.**

(ANS: $x=1, y=2, z=3, w=4$**)**

Q2. Find the value of *x, y ,a,* and *b* if

$$\begin{bmatrix} 2x-3y & a-b & 3 \\ 1 & x+4y & 3a+4b \end{bmatrix} = \begin{bmatrix} 1 & -2 & 3 \\ 1 & 6 & 29 \end{bmatrix}$$

(ANS: $x=2, y=1, a=3, b=5$**)**

Q3. For what values of x and y are the following matrices equal?

$$A = \begin{bmatrix} 2x+1 & 2y \\ 0 & y^2-5y \end{bmatrix}, \quad B = \begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$$

Q4. If $\begin{bmatrix} x+3 & z+4 & 2y-7 \\ 4x+6 & a-1 & 0 \\ b-3 & 3b & z+2c \end{bmatrix} = \begin{bmatrix} 0 & 6 & 3y-2 \\ 2x & -3 & 2c-2 \\ 2b+4 & -21 & 0 \end{bmatrix}$**.Obtain the values of *a, b, c, x, y,* and *z.***

(ANS: $a=-2, b=-7, c=-1, x=-3, y=-5, z=2$**)**

Q5. Give an example of

(i) a row matrix which is also a column matrix,

(ii) a diagonal matrix which is not scalar,

(iii) a triangular matrix

Q6. Construct a 2×3 **matrix whose elements** a_{ij} **are given by**

(i) $a_{ij} = \frac{(i+j)^2}{2}$ **(ii)** $a_{ij} = \frac{(i-j)^2}{2}$ **(iii)** $a_{ij} = \frac{(i-2j)^2}{2}$ **(iv)** $\frac{|2i-3j|}{2}$

(v) $a_{ij} = \frac{|-3i+j|}{2}$

Q7. Construct a 4×3 **matrix whose elements** a_{ij} **are given by**

(i) $a_{ij} = \frac{i-j}{i+j}$ **(ii)** $a_{ij} = i$ **(iii)** $a_{ij} = 2i + \frac{i}{j}$

(ANS: $\begin{bmatrix} 0 & -1/3 & -1/2 \\ 1/3 & 0 & -1/5 \\ 1/2 & 1/5 & 0 \\ 3/5 & 1/3 & 1/7 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \\ 4 & 4 & 4 \end{bmatrix}, \begin{bmatrix} 3 & 5/2 & 7/3 \\ 6 & 5 & 14/3 \\ 9 & 15/2 & 7 \\ 12 & 10 & 28/3 \end{bmatrix}$**)**

Q8. Find x, y, z ,t if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

(ANS: $x = 3, z = 9, y = 6 \text{ and } t = 6$**)**

Q9. Solve the matrix equation $\begin{bmatrix} x^2 \\ y^2 \end{bmatrix} - 3\begin{bmatrix} x \\ 2y \end{bmatrix} = \begin{bmatrix} -2 \\ 9 \end{bmatrix}$ **.**

(ANS: $x = 1, 2$ **and** $y = 3 \pm 3\sqrt{2}$ **.)**

Q10. Find matrices X and Y, if $2X - Y = \begin{bmatrix} 6 & -6 & 0 \\ -4 & 2 & 1 \end{bmatrix}$ **and**

$X + 2Y = \begin{bmatrix} 3 & 2 & 5 \\ -2 & 1 & -7 \end{bmatrix}$**.**

(ANS: $X = \begin{bmatrix} 3 & -2 & 1 \\ -2 & 1 & -1 \end{bmatrix}, Y = \begin{bmatrix} 0 & 2 & 2 \\ 0 & 0 & -3 \end{bmatrix}$**)**

Q11. Prove that the product of matrices

$\begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$ **and** $\begin{bmatrix} \cos^2\phi & \cos\phi\sin\phi \\ \cos\phi\sin\phi & \sin^2\phi \end{bmatrix}$ **is the null matrix when** θ and ϕ **differ by an odd multiple of** $\frac{\pi}{2}$**.**

Q12. If $A = \begin{bmatrix} 1 & -1 \\ 2 & -1 \end{bmatrix}, B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$ **and** $(A + B)^2 = A^2 + B^2$**, find a and b.**

(ANS: $a = 1, b = 4$**)**

Q13. If $A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$**, find x and y such that** $(xI + yA)^2 = A$**.**

(ANS: $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$ **or** $\left(\frac{i}{\sqrt{2}}, -\frac{i}{\sqrt{2}}\right)$ or $\left(-\frac{i}{\sqrt{2}}, \frac{i}{\sqrt{2}}\right)$**)**

Q14. Let $A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$**,** $B = \begin{bmatrix} 5 & 2 \\ 7 & 4 \end{bmatrix}$**,** $C = \begin{bmatrix} 2 & 5 \\ 3 & 8 \end{bmatrix}$**. Find a matrix** D **such that** $CD - AB = 0$**.**

(ANS: $\begin{bmatrix} -191 & -110 \\ 77 & 44 \end{bmatrix}$**)**

Q15. Find the value of 'x' such that:

$$\begin{bmatrix} 1 & x & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 2 \\ 2 & 5 & 1 \\ 15 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ x \end{bmatrix} = 0.$$

Q16. If $\begin{bmatrix} 2 & -1 \\ 1 & 0 \\ -3 & 4 \end{bmatrix} A = \begin{bmatrix} -1 & -8 & -10 \\ 1 & -2 & -5 \\ 9 & 22 & 15 \end{bmatrix}$**, find A.**

(ANS: $\begin{bmatrix} 1 & -2 & -5 \\ 3 & 4 & 0 \end{bmatrix}$**)**

Q17. Let $f(x) = x^2 - 5x + 6$**.Find** $f(A)$ **if** $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$**.**

(ANS: $\begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$**)**

Q18. Let $A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$ **and** $f(x) = x^2 - 4x + 7$**.Show that** $f(A) = O$**.Use this result to find** A^5**.**

(ANS: $A^5 = \begin{bmatrix} -118 & -93 \\ 31 & -118 \end{bmatrix}$**)**

Q19. Prove the following by the principle of mathematical induction:

If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$**, then** $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$ **for every positive integer n.**

Q20. If $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$**, then prove that**

(i) $A_\alpha . A_\beta = A_{\alpha+\beta}$ **(ii)** $(A_\alpha)^n = \begin{bmatrix} \cos n\alpha & \sin n\alpha \\ -\sin n\alpha & \cos n\alpha \end{bmatrix}$**, for every positive integer n.**

Q21. If 'a' is a non-zero real or complex number. Use the principle of mathematical induction to prove that

If $A = \begin{bmatrix} a & 1 \\ 0 & a \end{bmatrix}$**, then** $A^n = \begin{bmatrix} a^n & na^{n-1} \\ 0 & a^n \end{bmatrix}$ **for every positive integer n.**

Q22. Under what condition is the matrix equation $A^2 - B^2 = (A-B)(A+B)$ **is true?**

Q23. If $AB = A$ **and** $BA = B$**, then show that** $A^2 = A, B^2 = B$**.**

Q24.If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & -2 \\ a & 2 & b \end{bmatrix}$ **is a matrix satisfying** $AA^T = 9I_3$**, then find the values of 'a' and 'b'.**

(ANS: $a = -2$ **and** $b = -1$**)**

Q25. Find the values of x, y, z if the matrix $A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$ **satisfy the equation** $A^T A = I_3$**.**

(ANS: $x = \pm\frac{1}{\sqrt{2}}, y = \pm\frac{1}{\sqrt{6}}, z = \pm\frac{1}{\sqrt{3}}$**)**

ASSIGNMENT - DETERMINANTS

Show that each one of the following systems of equations is inconsistent.

1. $x + 2y = 9;$
   $2x + 4y = 7.$

2. $2x + 3y = 5;$
   $6x + 9y = 10.$

3. $4x - 2y = 3;$
   $6x - 3y = 5.$

4. $6x + 4y = 5;$
   $9x + 6y = 8.$

5. $x + y - 2z = 5;$
   $x - 2y + z = -2;$
   $-2x + y + z = 4.$

6. $2x - y + 3z = 1;$
   $3x - 2y + 5z = -4;$
   $5x - 4y + 9z = 14.$

7. $x + 2y + 4z = 12;$
   $y + 2z = -1;$
   $3x + 2y + 4z = 4.$

8. $3x - y - 2z = 2;$
   $2y - z = -1;$
   $3x - 5y = 3.$

Solve each of the following systems of equations using matrix method.

9. $5x + 2y = 4;$
   $7x + 3y = 5.$

10. $3x + 4y - 5 = 0;$
    $x - y + 3 = 0.$

11. $x + 2y = 1;$
    $3x + y = 4.$

12. $5x + 7y + 2 = 0;$
    $4x + 6y + 3 = 0.$

13. $2x - 3y + 1 = 0;$
    $x + 4y + 3 = 0.$

14. $4x - 3y = 3;$
    $3x - 5y = 7.$

15. $2x + 8y + 5z = 5;$
    $x + y + z = -2;$
    $x + 2y - z = 2.$ **[CBSE 2009C]**

16. $x - y + z = 1;$
    $2x + y - z = 2;$
    $x - 2y - z = 4.$ **[CBSE 2006C]**

17. $3x + 4y + 7z = 4;$
    $2x - y + 3z = -3;$
    $x + 2y - 3z = 8.$ **[CBSE 2012]**

18. $x + 2y + z = 7;$
    $x + 3z = 11;$
    $2x - 3y = 1.$ **[CBSE 2005, '08, '11]**

19. $2x - 3y + 5z = 16;$
    $3x + 2y - 4z = -4;$
    $x + y - 2z = -3.$ **[CBSE 2005C]**

20. $x + y + z = 4;$
    $2x - y + z = -1;$
    $2x + y - 3z = -9.$ **[CBSE 2005]**

21. $2x - 3y + 5z = 11;$
    $3x + 2y - 4z = -5;$
    $x + y - 2z = -3.$ **[CBSE 2009]**

22. $x + y + z = 1;$
    $x - 2y + 3z = 2;$
    $5x - 3y + z = 3.$ **[CBSE 2004, '09C]**

23. $x + y + z = 6;$
    $x + 2z = 7;$
    $3x + y + z = 12.$ **[CBSE 2009]**

24. $2x + 3y + 3z = 5;$
    $x - 2y + z = -4;$
    $3x - y - 2z = 3.$ **[CBSE 2008C, '12]**

25. $4x - 5y - 11z = 12;$
    $x - 3y + z = 1;$
    $2x + 3y - 7z = 2.$ **[CBSE 2007]**

26. $x - y + 2z = 7;$
    $3x + 4y - 5z = -5;$
    $2x - y + 3z = 12.$ **[CBSE 2012]**

27. $6x - 9y - 20z = -4;$
    $4x - 15y + 10z = -1;$
    $2x - 3y - 5z = -1.$

28. $3x - 4y + 2z = -1;$
    $2x + 3y + 5z = 7;$
    $x + z = 2.$ **[CBSE 2011C]**

29. $x + y - z = 1;$
    $3x + y - 2z = 3;$
    $x - y - z = -1.$ **[CBSE 2004]**

30. $2x + y - z = 1;$
    $x - y + z = 2;$
    $3x + y - 2z = -1.$ **[CBSE 2004C]**

31. $x + 2y + z = 4;$
$-x + y + z = 0;$
$x - 3y + z = 4.$ **[CBSE 2012C]**

32. $x - y - 2z = 3;$
$x + y = 1;$
$x + z = -6.$

33. $5x - y = -7;$
$2x + 3z = 1;$
$3y - z = 5.$

34. $x - 2y + z = 0;$
$y - z = 2;$
$2x - 3z = 10.$

35. $x - y = 3;$
$2x + 3y + 4z = 17;$
$y + 2z = 7.$ **[CBSE 2003C, '07C]**

36. $4x + 3y + 2z = 60;$
$x + 2y + 3z = 45;$
$6x + 2y + 3z = 70.$ **[CBSE 2011]**

37. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. **[CBSE 2007C, '08C]**

Using A^{-1}, solve the following system of equations:
$2x - 3y + 5z = 11;$
$3x + 2y - 4z = -5;$
$x + y - 2z = -3.$

38. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of linear equations:
$2x + y + z = 1;$
$x - 2y - z = \frac{3}{2};$
$3y - 5z = 9.$

HINT: Here $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$.

39. If $A = \begin{bmatrix} 1 & -2 & 0 \\ 2 & 1 & 3 \\ 0 & -2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 7 & 2 & -6 \\ -2 & 1 & -3 \\ -4 & 2 & 5 \end{bmatrix}$, find AB.

Hence, solve the system of equations:
$x - 2y = 10,\ 2x + y + 3z = 8$ and $-2y + z = 7.$ [CBSE 2011]

HINT: $AB = (11)I \Rightarrow A\left(\frac{1}{11}B\right) = I \Rightarrow A^{-1} = \left(\frac{1}{11}\right)B.$

Using matrices, solve the following system of equations.

40. $\frac{2}{x} - \frac{3}{y} + \frac{3}{z} = 10,\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 10,\ \frac{3}{x} - \frac{1}{y} + \frac{2}{z} = 13$ [CBSE 2007C]

41. $\frac{1}{x} - \frac{1}{y} + \frac{1}{z} = 4;\ \frac{2}{x} + \frac{1}{y} - \frac{3}{z} = 0;\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 2\ (x, y, z \neq 0)$

ASSIGNMENT – XII – INVERSE TRIGONOMETRIC FUNCTIONS

Following questions carry 1 marks each

1. Evaluate: $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.
2. Show that $\sin^{-1}\left(2x\sqrt{1-x^2}\right)=2\sin^{-1}x$
3. Write the domain of the function $\cos ec^{-1}x$.
4. Write the value of $\sin\left[\frac{\pi}{3}-\sin^{-1}\left(-\frac{1}{2}\right)\right]$.
5. Write the principal value of $\cos^{-1}\left(\frac{1}{2}\right)-2\sin^{-1}\left(-\frac{1}{2}\right)$.
6. Write $\cot^{-1}\left(\frac{1}{\sqrt{x^2-1}}\right), |x|>1$ in simplest form.
7. Write the value of $\sin\left(2\sin^{-1}\frac{3}{5}\right)$ and $\tan\left(2\tan^{-1}\frac{1}{5}\right)$.
8. Write the value of $\tan^{-1}\left[2\sin\left(2\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)\right)\right]$.
9. If $\tan^{-1}x+\tan^{-1}y=\frac{\pi}{4},\ xy<1$, then write the value of $x+y+xy$.
10. If $\sin\left(\sin^{-1}\frac{1}{5}+\cos^{-1}x\right)=1$, then find the value of ‘x’.

Following questions carry 4 marks each

11. Prove that: $2\tan^{-1}\left(\frac{1}{5}\right)+\sec^{-1}\left(\frac{5\sqrt{2}}{7}\right)+2\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.
12. Prove that: $\tan^{-1}\left[\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right]=\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x,\ \ -\frac{1}{\sqrt{2}}\le x\le 1$.
13. Prove that: $\cot^{-1}7+\cot^{-1}8+\cot^{-1}18=\cot^{-1}3$.
14. Prove that: $\tan^{-1}\left(\frac{1}{2}\right)+\tan^{-1}\left(\frac{1}{5}\right)\tan^{-1}\left(\frac{1}{8}\right)=\frac{\pi}{4}$.
15. Show that: $\tan\left[\frac{1}{2}\sin^{-1}\left(\frac{3}{4}\right)\right]=\frac{4-\sqrt{7}}{3}$.
16. If $y=\cot^{-1}\left(\sqrt{\cos x}\right)-\tan^{-1}\left(\sqrt{\cos x}\right)$, then prove that $\sin y=\tan^2\left(\frac{x}{2}\right)$.

17. Prove that: $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right)=\frac{\pi}{4}-\frac{x}{2},\ x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$.

18. Prove that: $\sin^{-1}\left(\frac{8}{17}\right)+\sin^{-1}\left(\frac{3}{5}\right)=\cos^{-1}\left(\frac{36}{85}\right)$.

19. Prove that: $\cos\left(\sin^{-1}\frac{3}{5}+\cot^{-1}\frac{3}{2}\right)=\frac{6}{5\sqrt{13}}$.

20. Prove that: $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3=\pi$.

21. Prove that: $\cos\left[\tan^{-1}\left\{\sin\left(\cot^{-1}x\right)\right\}\right]=\sqrt{\frac{1+x^2}{2+x^2}}$.

22. Prove that: $\tan\left[\frac{\pi}{4}+\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]+\tan\left[\frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{a}{b}\right)\right]=\frac{2b}{a}$.

23. Prove that: $\sin^{-1}\left(\frac{4}{5}\right)+\sin^{-1}\left(\frac{5}{13}\right)+\sin^{-1}\left(\frac{16}{65}\right)=\frac{\pi}{2}$.

24. Solve: $\tan^{-1}\left(\frac{1-x}{1+x}\right)=\frac{1}{2}\tan^{-1}x$; $x>0$. (Ans: $\frac{1}{\sqrt{3}}$)

25. Solve : $\tan^{-1}(x+1)+\tan^{-1}(x-1)=\tan^{-1}\left(\frac{8}{31}\right)$. (Ans: $\frac{1}{4}$)

26. Solve: $\cos^{-1}\left(\frac{x^2-1}{x^2+1}\right)+\tan^{-1}\left(\frac{2x}{x^2-1}\right)=\frac{2\pi}{3}$. (Ans: $\frac{\sqrt{3}-1}{\sqrt{3}+1}$)

27. Solve: $2\tan^{-1}(\sin x)=\tan^{-1}(2\sec x)$, where $x\neq\frac{\pi}{2}$ (Ans: $\frac{\pi}{4}$)

28. Solve: $\cos\left(\tan^{-1}x\right)=\sin\left(\cot^{-1}\frac{3}{4}\right)$. (Ans: $\pm\frac{3}{4}$)

29. Solve: $\cot^{-1}x-\cot^{-1}(x+2)=\frac{\pi}{12}$ (Ans: $\sqrt{3}$)

30. Solve: $\cos\left[2\sin^{-1}(-x)\right]=0$ (Ans: $x=\pm\frac{1}{\sqrt{2}}$)

31. Solve: $\sin^{-1}(1-x)+\sin^{-1}x=\cos^{-1}x$ (Ans: $x=0,\frac{1}{2}$)

32. Solve: $\sin^{-1}\left(6\sqrt{3}x\right)+\sin^{-1}(6x)=\frac{\pi}{2}$ (Ans: $x=\pm\frac{1}{12}$)

33. Solve: $\sin^{-1}\frac{15}{x}+\sin^{-1}\frac{8}{x}=\frac{\pi}{2}$. (Ans: $x=\pm17$)

34. Solve: $\tan^{-1}\left(\frac{x+1}{x-1}\right)+\tan^{-1}\left(\frac{x-1}{x}\right)=-\tan^{-1}7$ (Ans: 2)

Example 11. *If a young man rides his motor cycle at 25 km per hour, he has to spend ₹ 2 per kilometre on petrol; if he rides at a faster speed of 40 km per hour, the petrol cost increases to ₹ 5 per kilometre. He has ₹ 100 to spend on petrol and wishes to find the maximum distance he can travel within one hour. Express this as a linear programming problem and then solve it.* (C.B.S.E. 2007)

Solution. Let x km and y km be the distances covered by the young man at the speeds of 25 km/hr and 40 km/hr respectively, then time consumed in covering these distance are $\frac{x}{25}$ hr and $\frac{y}{40}$ hr respectively. Total distance travelled by the young man $D = x + y$ (kilometres).

Hence, the problem can be formulated as an L.P.P. as follows :

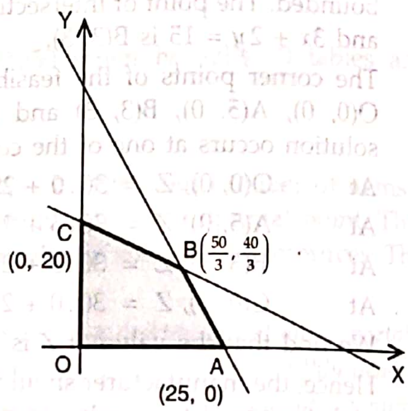
Maximize $D = x + y$ subject to the constraints

$2x + 5y \le 100$ (money constraint)

$\frac{x}{25} + \frac{y}{40} \le 1$ *i.e.* $8x + 5y \le 200$ (time constraint)

$x \ge 0, y \ge 0$ (non-negative constraints)

We draw the straight lines $2x + 5y = 100$, $8x + 5y = 200$ and shade the region satisfied by the above inequalities. The shaded portion shows the feasible region OABC which is bounded. The point of intersection of the lines is $B\left(\frac{50}{3}, \frac{40}{3}\right)$.



The corner points of the feasible region OABC are O(0, 0), A(25, 0), $B\left(\frac{50}{3}, \frac{40}{3}\right)$ and C(0, 20).

The optimal solution occurs at one of the corner points.

At O(0, 0), $D = 0 + 0 = 0$.

At A(25, 0), $D = 25 + 0 = 25$.

At $B\left(\frac{50}{3}, \frac{40}{3}\right)$, $D = \frac{50}{3} + \frac{40}{3} = 30$.

At C(0, 20), $D = 0 + 20 = 20$.

We find that the value of D is maximum at $B\left(\frac{50}{3}, \frac{40}{3}\right)$.

Hence, the young man covers a total distance of 30 km, $\frac{50}{3}$ km at 25 km/hr and $\frac{40}{3}$ km at 40 km/hr.

Example 12. *A manufacturer makes ₹ 600 profit on each 21" TV set it produces and ₹ 400 profit on each 14" TV set. A 21" TV requires 1 hour on machine X, 1 hour on machine Y and 4 hours on machine Z. The 14" TV requires 2 hours on X, 1 hour on Y and 1 hour on Z. In a given day, machines X, Y and Z can work a maximum of 16, 9 and 24 hours respectively. How many 21" TV sets and how many 14" TV sets should be produced per day to maximize the profit?*

Solution. Let x be the number of 21" TV sets and y be the number of 14" TV sets produced (and sold). Then the problem can be formulated as :

Maximize $f(x, y) = 600x + 400y$ subject to the constraints

$x + 2y \le 16$ (machine X constraint)

$x + y \le 9$ (machine Y constraint)

$4x + y \le 24$ (machine Z constraint)

$x \ge 0, y \ge 0$ (number of TV sets cannot be negative)

CHAPTER TEST

1. *The corner points of the feasible region determined by the following systems of linear inequalities :*
   $2x + y \leq 10$, $x + 3y \leq 15$, $x \geq 0$, $y \geq 0$ are (0, 0), (5, 0), (3, 4) and (0, 5).
   Let $Z = px + qy$, when $p, q > 0$, then find the relation between p and q so that the maximum of Z occurs at both points (3, 4) and (0, 5). *(NCERT)*

2. A producer has 30 and 17 units of labour and capital respectively which he can use to produce two types of goods X and Y. To produce one unit of X, 2 units of labour and 3 units of capital are required. Similarly, 3 units of labour and 1 unit of capital are required to produce one unit of Y. If X and Y are priced at ₹100 and ₹120 per unit respectively, how should the producer use his resources to maximize the total revenue? Solve the problem graphically. *(C.B.S.E. 2000)*

3. Suppose every gram of wheat provides 0.1 g of proteins and 0.25 g of carbohydrates, and the corresponding values for rice are 0.05 g and 0.5 g respectively. Wheat costs ₹20 and rice ₹30 per kilogram. The minimum daily requirement of an average man for proteins and carbohydrates is 50 g and 200 g respectively. In what quantities should wheat and rice be mixed in the daily diet to provide the minimum daily requirements of proteins and carbohydrates at minimum cost? What is the minimum cost? *(C.B.S.E. Sample Paper)*

4. A man owns a field of area 1000 sq. metre. He wants to plant trees in it. He has a sum of ₹1400 to purchase young trees. He has the choice of two types of trees. Tree of type A requires 10 sq. metre of ground per tree and costs ₹20 per tree and type B requires 20 sq metre of ground per tree and costs ₹25 per tree. When fully grown, type A produces an average of 20 kg of fruits which can be sold at a profit of ₹2 per kg and type B produces an average of 40 kg of fruits which can be sold at a profit of ₹1.50 per kg. How many trees of each type should be planted to achieve maximum profit when the trees are fully grown? What is maximum profit?

5. A company sells two different products, A and B. The two products are produced in a common production process, which has a total capacity of 500 man-hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. A market survey shows that maximum 70 units of A and 125 units of B can be sold. If the profit is ₹20 per unit for the product A and ₹15 per unit for the product B, how many units of each product should be sold to maximize profit? *(C.B.S.E. 2003)*

6. A small manufacturer has employed 5 skilled men and 10 semi-skilled men and makes an article in two qualities—a deluxe model and an ordinary model. The making of a deluxe model requires 2 hours of work by a skilled man and 2 hours of work by a semi-skilled man. The ordinary model requires 1 hour by a skilled man and 3 hours by a semi-skilled man. By union rule, no man may work more than 8 hours a day. The manufacturer gains ₹15 on deluxe model and ₹10 on ordinary model. How many of each type should be made in order to maximize his total daily profit?

7. A toy company manufactures two types of dolls, A and B. Each doll of type B takes twice as long to produce as one of type A. If the company produces only type A, it can make a maximum of 2000 dolls per day. The supply of plastic is sufficient to produce 1500 dolls per day. Type B requires a fancy dress which cannot be available for more than 600 dolls per day. If the company makes profits of ₹30 and ₹50 per doll respectively on dolls A and B, how many of each should be produced per day in order to maximize the profit?

8. A company manufactures two articles A and B. There are two departments through which these articles are processed : (*i*) assembly, and (*ii*) finishing departments. The maximum capacity of the first department is 60 hours a week and that of the other department is 48 hours a week. The production of each article A requires 4 hours in assembly and 2 hours in finishing and that of each unit of B requires 2 hours in assembly and 4 hours in finishing. If the profit is ₹60 for each unit of A and ₹80 for each

EXAMPLE 6 *A resourceful home decorator manufactures two types of lamps say A and B. Both lamps go through two technicians, first a cutter, second a finisher. Lamp A requires 2 hours of the cutter's time and 1 hour of the finisher's time. Lamp B requires 1 hour of cutter's and 2 hours of finisher's time. The cutter has 104 hours and finisher has 76 hours of time available each month. Profit on one lamp A is Rs 6.00 and on one lamp B is Rs 11.00. Assuming that he can sell all that he produces, how many of each type of lamps should he manufacture to obtain the best return.*

SOLUTION The above information can be put in the following tabular form :

Lamp	*Cutter's time*	*Finisher's time*	*Profit in Rs*
A	2	1	6
B	1	2	11
Maximum time available	104	76	

Let the decorator manufacture x lamps of type A and y lamps of type B.

$\therefore$ Total profit $=$ Rs $(6x + 11y)$

Total time taken by the cutter in preparing x lamps of type A and y lamps of type B is $(2x + y)$ hours. But the cutter has 104 hours only for each month.

$\therefore$ $2x + y \leq 104.$

Similarly, the total time taken by the finisher in preparing x lamps of type A and y lamps of type B is $(x + 2y)$ hours. But the cutter has 76 hours only for each month.

$\therefore$ $x + 2y \leq 76.$

Since the number of lamps cannot be negative.

$\therefore$ $x \geq 0$ and $y \geq 0.$

Let Z denote the total profit. Then, $Z = 6x + 11y$.

Since the profit is to be maximized. So, the mathematical formulation of the given LPP is as follows :

Maximize $Z = 6x + 11y$

Subject to

$2x + y \leq 104$

$x + 2y \leq 76$

and, $x \geq 0 , y \geq 0$

EXAMPLE 7 *A company makes two kinds of leather belts, A and B. Belt A is high quality belt, and B is of lower quality. The respective profits are Rs 4 and Rs 3 per belt. Each belt of type A requires twice as much time as a belt of type B, and if all belts were of type B, the company could make 1000 belts per day. The supply of leather is sufficient for only 800 belts per day (both A and B combined). Belt A requires a fancy buckle, and only 400 buckles per day are available. There are only 700 buckles available for belt B. What should be the daily production of each type of belt ? Formulate the problem as a LPP.*

SOLUTION Suppose the company makes per day x belts of type A and y belts of type B.

$\therefore$ Profit $= 4x + 3y.$

Let Z denote the profit. Then, $Z = 4x + 3y$ and it is to be maximized.

It is given that 1000 belts of type B can be made per day and each belt of type A requires twice as much time as a belt of type B. So, 500 belts of type A can be made in a day. So, total time taken in preparing x belts of type A and y belts of type B is $\left(\frac{x}{500}+\frac{y}{1000}\right)$. But the company is making x belts of type A and y belts of type B in a day.

$$\therefore \quad \frac{x}{500}+\frac{y}{1000} \leq 1 \Rightarrow 2x+y \leq 1000.$$

Since the supply of leather is sufficient for only 800 belts per day.

$$\therefore \quad x+y \leq 800.$$

It is given that only 400 fancy buckles for type A and 700 buckles for type B are available per day.

$$\therefore \quad x \leq 400,\ y \leq 700.$$

Finally, the number of belts cannot be negative.

$$\therefore \quad x \geq 0 \text{ and } y \geq 0.$$

Thus, the mathematical formulation of the given LPP is as follows :

Maximize $Z = 4x+3y$

Subject to

$$2x+y \leq 1000$$
$$x+y \leq 800$$
$$x \leq 400$$
$$y \leq 700$$

and, $x \geq 0,\ y \geq 0.$

Type II **DIET PROBLEMS**

EXAMPLE 8 *A dietician whishes to mix two types of food in such a way that the vitamin contents of the mixture contain at least 8 units of Vitamin A and 10 units of vitamin C. Food 'I' contains 2 units per kg of vitamin A and 1 unit per kg of vitamin C while food 'II' contains 1 unit per kg of vitamin A and 2 units per kg of vitamin C. It costs Rs 5.00 per kg to purchase food 'I' and Rs 7.00 per kg to produce food 'II'. Formulate the above linear programming problem to minimize the cost of such a mixture.*

SOLUTION The gives data may be put in the following tabular form :

Resources	*Food*		*Requirements*
	I	II	
Vitamin A	2	1	8
Vitamin B	1	2	10
Cost (in Rs)	5	7	

Let the dietician mix x kg of food 'I' and y kg of food 'II'.

Clearly, $x \geq 0, y \geq 0$.

Since one kg of food 'I' costs Rs 5 and one kg of food 'II' costs Rs 7. Therefore, total cost of x kg of food 'I' and y kg of food 'II' is Rs $(5x+7y)$.

$x + 3y \le 90$

and, $x \ge 0,\ y \ge 0$

EXAMPLE 5 *A manufacturer of a line of patent medicines is preparing a production plan on medicines A and B. There are sufficient ingredients available to make 20,000 bottles of A and 40,000 bottles of B but there are only 45,000 bottles into which either of the medicines can be put. Further more, it takes 3 hours to prepare enough material to fill 1000 bottles of A, it takes one hour to prepare enough material to fill 1000 bottles of B and there are 66 hours available for this operation. The profit is Rs 8 per bottle for A and Rs 7 per bottle for B. Formulate this problem as a linear programming problem.*

SOLUTION Suppose the manufacturer produces x bottles of medicines A and y bottles of medicine B.

Since the profit is Rs 8 per bottle for A and Rs 7 per bottle for B. So, total profit in producing x bottles of medicine A and y bottles of medicine B is Rs $(8x + 7y)$.

Let Z denote the total profit. Then,

$$Z = 8x + 7y$$

Since 1000 bottles of medicine A are prepared in 3 hours. So,

Time required to prepare x bottles of medicine $A = \frac{3x}{1000}$ hours.

It is given that 1000 bottles of medicine B are prepared in 1 hour.

$\therefore$ Time required to prepare y bottles of medicine $B = \frac{y}{1000}$ hours.

Thus, total time required to prepare x bottles of medicine A and y bottles of medicine B is $\frac{3x}{1000} + \frac{y}{1000}$ hours. But, the total time available for this operation is 66 hours.

$$\therefore \quad \frac{3x}{1000} + \frac{y}{1000} \le 66$$

$$\Rightarrow \quad 3x + y \le 66000$$

Since there are only 45,000 bottles into which the medicines can be put.

$$\therefore \quad x + y \le 45{,}000$$

It is given that the ingredients are available for 20,000 bottles of A and 40,000 bottles of B.

$$\therefore \quad x \le 20{,}000 \text{ and } y \le 40{,}000$$

Since the number of bottles can not be negative. Therefore, $x \ge 0, y \ge 0$.

Hence, the mathematical formulation of the given LPP is as follows :

Maximize $Z = 8x + 7y$

Subject to

$3x + y \le 66{,}000$

$x + y \le 45{,}000$

$x \le 20{,}000$

$y \le 40{,}000$

and, $x > 0, y \ge 0.$

Suppose x units of product P_1 and y units of product P_2 are produced to maximize the profit. Let Z denote the total profit.

Since each unit of product P_1 requires 2 hrs for moulding and each unit of product P_2 requires 4 hrs for moulding. Hence, the total hours required for moulding for x units of product P_1 and y units of product P_2 are $2x + 4y$. This must be less than or equal to the total hours available for moulding. Hence,

$$2x + 4y \leq 20$$

This is the first constraint.

The total hours required for grinding for x units of product P_1 and y units of product P_2 is $3x + 2y$. But the maximum number of hours available for grinding is 24.

$$\therefore \quad 3x + 2y \leq 24$$

This is the second constraint.

Similarly for polishing the constraint is $4x + 2y \leq 13$.

Since x and y are non-negative integers, therefore $x \geq 0,\ y \geq 0$

The total profit for x units of product P_1 and y units of product P_2 is $5x + 3y$. Since we wish to maximize the profit, therefore the objective function is

$$\text{Maximize} \quad Z = 5 + 3y$$

Hence, the linear programming problem for the given problem is as follows

$$\textit{Maximize} \quad Z = 5x + 3y$$

Subject to the constraints

$$2x + 4y \leq 20,$$

$$3x + 2y \leq 24,$$

$$4x + 2y \leq 13,$$

and, $x \geq 0,\ y \geq 0$

EXAMPLE 2 *A toy company manufactures two types of doll; a basic version doll A and a deluxe version doll B. Each doll of type B takes twice as long to produce as one of type A, and the company would have time to make a maximum of 2,000 per day if it produces only the basic version. The supply of plastic is sufficient to produce 1500 dolls per day (both A and B combined). The deluxe version requires a fancy dress of which there are only 600 per day available. If the company makes profit of Rs 3 and Rs 5 per doll respectively on doll A and doll B; how many of each should be produced per day in order to maximize profit?*

SOLUTION Let x dolls of type A and y dolls of type B be produced per day. Then,

$$\text{Total profit} = 3x + 5y.$$

Since each doll of type B takes twice as long to produce as one of type A, therefore total time taken to produce x dolls of type A and y dolls of type B is $x + 2y$. But the company has time to make a maximum of 2000 dolls per day

$$\therefore \quad x + 2y \leq 2000$$

Since plastic is available to produce 1500 dolls only.

$$\therefore \quad x + y \leq 1500$$

Also fancy dress is available for 600 dolls per day only

$$\therefore \quad y \leq 600$$